Numerical Methods in Hydrodynamics

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Challenges

- Coupling of Velocity and Pressure (Water Level)
- Convection Terms
- Wetting and Drying
- Irregular and Movable Domain

Goal: Model Stability, Efficiency and Reliability

Linkage between Velocity and Pressure in NS Equations

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0$$

$$\frac{\partial u_x}{\partial t}+\frac{\partial (u_x^2)}{\partial x}+\frac{\partial (u_y u_x)}{\partial y}+\frac{\partial (u_z u_x)}{\partial z}=\frac{1}{\rho}F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}$$

$$\frac{\partial u_y}{\partial t}+\frac{\partial (u_x u_y)}{\partial x}+\frac{\partial (u_y^2)}{\partial y}+\frac{\partial (u_z u_y)}{\partial z}=\frac{1}{\rho}F_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{yy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yz}}{\partial z}$$

$$\frac{\partial u_z}{\partial t}+\frac{\partial (u_x u_z)}{\partial x}+\frac{\partial (u_y u_z)}{\partial y}+\frac{\partial (u_z^2)}{\partial z}=\frac{1}{\rho}F_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{zy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{zz}}{\partial z}$$

Weak linkage: The momentum equations link the velocity to the pressure gradient, while the continuity equation is just an additional constraint on the velocity field without directly linking to the pressure.

3-D Shallow Water Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+\frac{\partial (u^2)}{\partial x}+\frac{\partial (vu)}{\partial y}+\frac{\partial (wu)}{\partial z}=-g\frac{\partial z_s}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}+f_c v$$

$$\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (v^2)}{\partial y}+\frac{\partial (wv)}{\partial z}=-g\frac{\partial z_s}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{yy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{yz}}{\partial z}-f_c u$$

Free-Surface Kinematic Condition

$$\frac{\partial z_s}{\partial t}+u_h\frac{\partial z_s}{\partial x}+v_h\frac{\partial z_s}{\partial y}=w_h$$

The system still keep the weak linkage in the full 3-D model, with the pressure being governed by a 2-D equation.

2-D Shallow Water Equations

$$\frac{\partial h}{\partial t}+\frac{\partial (hU)}{\partial x}+\frac{\partial (hV)}{\partial y}=0$$

$$\frac{\partial (hU)}{\partial t}+\frac{\partial (hUU)}{\partial x}+\frac{\partial (hVU)}{\partial y}=-gh\frac{\partial z_s}{\partial x}+\frac{1}{\rho}\frac{\partial (hT_{xx})}{\partial x}+\frac{1}{\rho}\frac{\partial (hT_{xy})}{\partial y}+\frac{1}{\rho}\left(\tau_{sx}-\tau_{bx}\right)+f_c hV$$

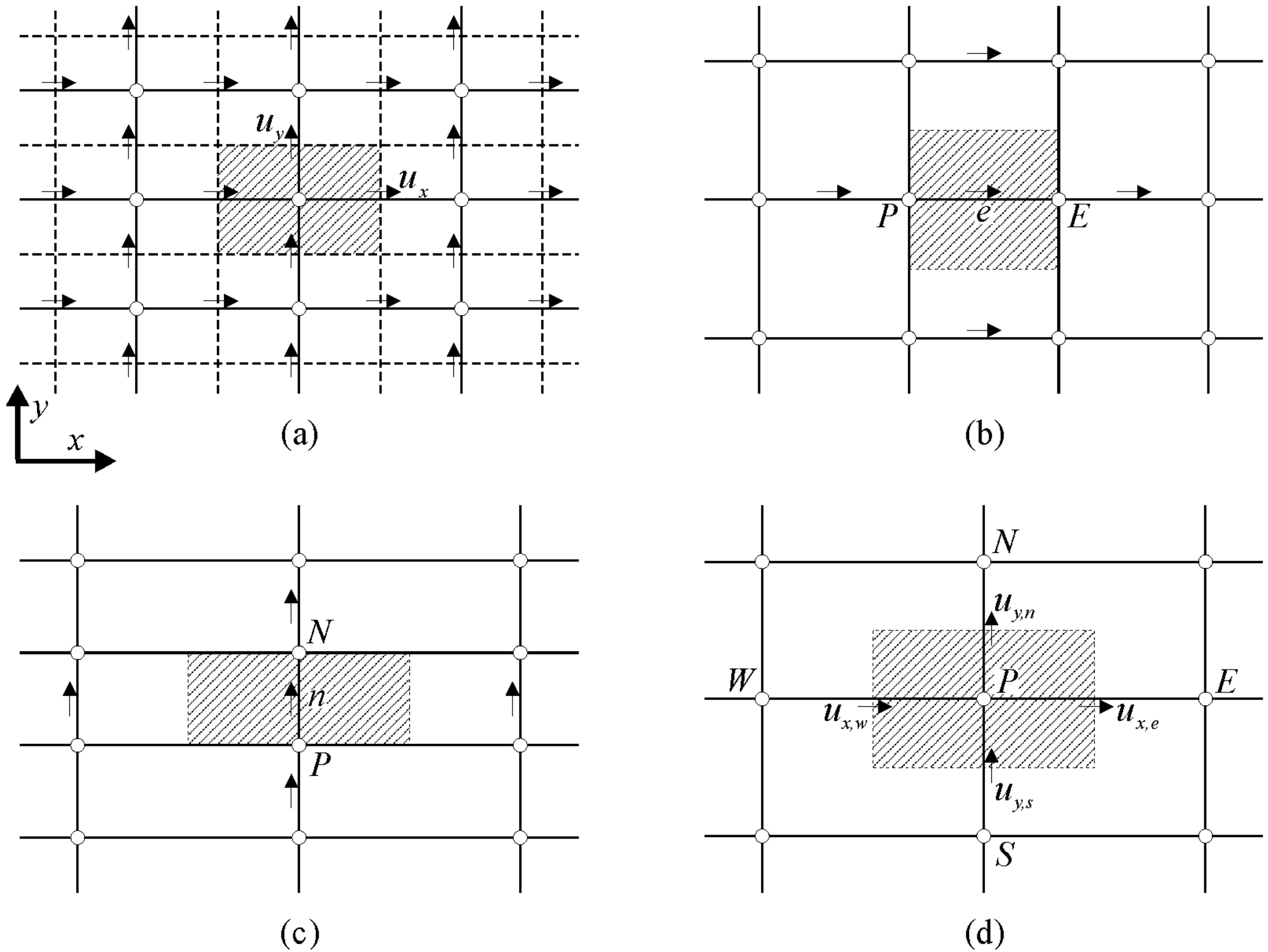
$$\frac{\partial (hV)}{\partial t}+\frac{\partial (hUV)}{\partial x}+\frac{\partial (hVV)}{\partial y}=-gh\frac{\partial z_s}{\partial y}+\frac{1}{\rho}\frac{\partial (hT_{yx})}{\partial x}+\frac{1}{\rho}\frac{\partial (hT_{yy})}{\partial y}+\frac{1}{\rho}\left(\tau_{sy}-\tau_{by}\right)-f_c hU$$

The linkage in the continuity equation is improved, but that in the momentum equations is still the same as in the Navier-Stokes equations.

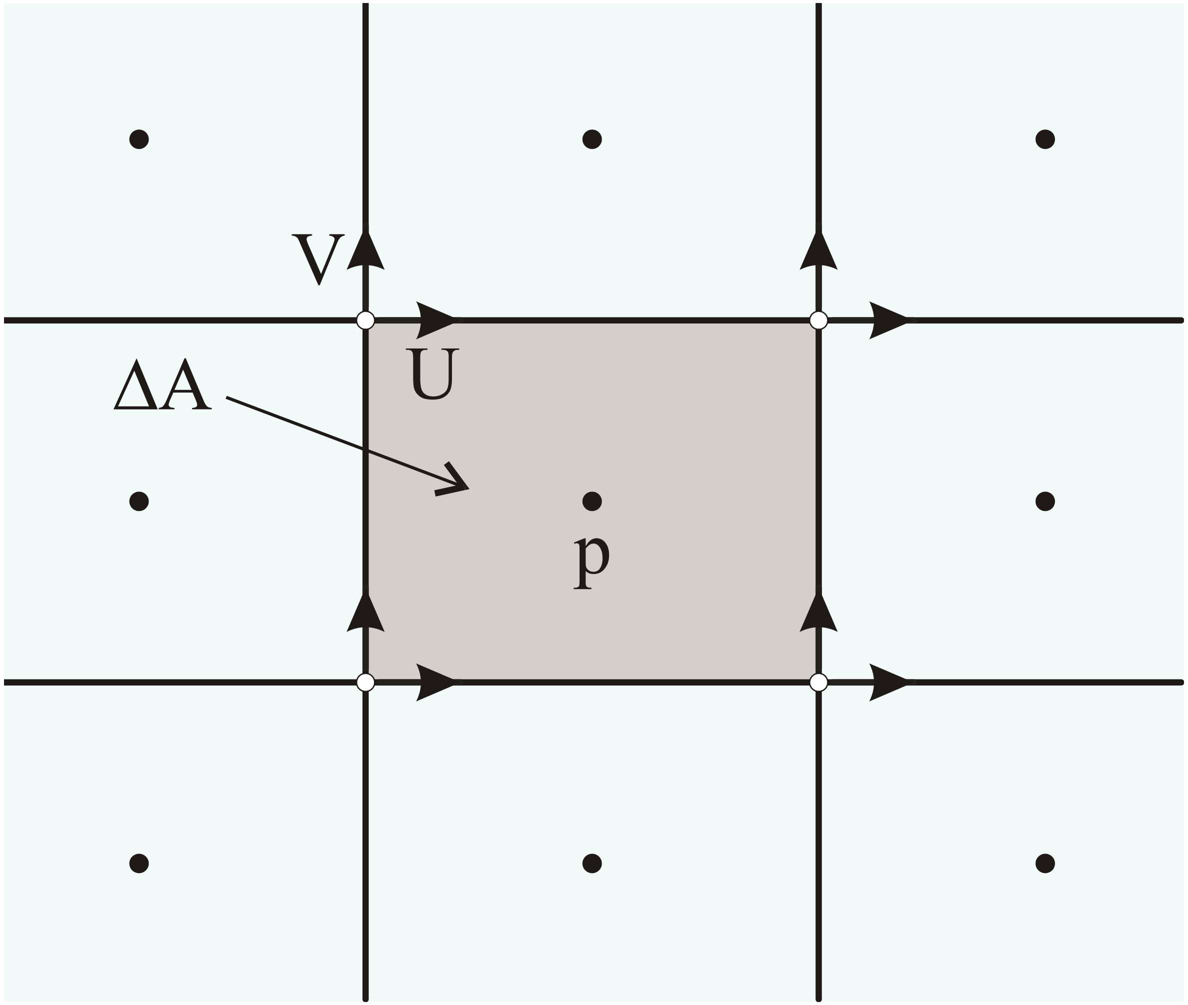
Velocity-Pressure Coupling

- MAC Algorithm (Harlow &Welch, 1965)
  - Staggered grid
- Projection Method (Chorin, 1968)
  - Staggered grid
- SIMPLE Algorithms – Staggered grid
  - SIMPLE (Patankar and Spalding, 1972)
  - SIMPLER (Patankar, 1980)
  - PISO (Issa, 1982)
  - SIMPLEC (van Doormaal & Raithby, 1984)
- SIMPLE(C) – Non-staggered grid
  - (Rhie and Chow, 1983; Peric, 1985)

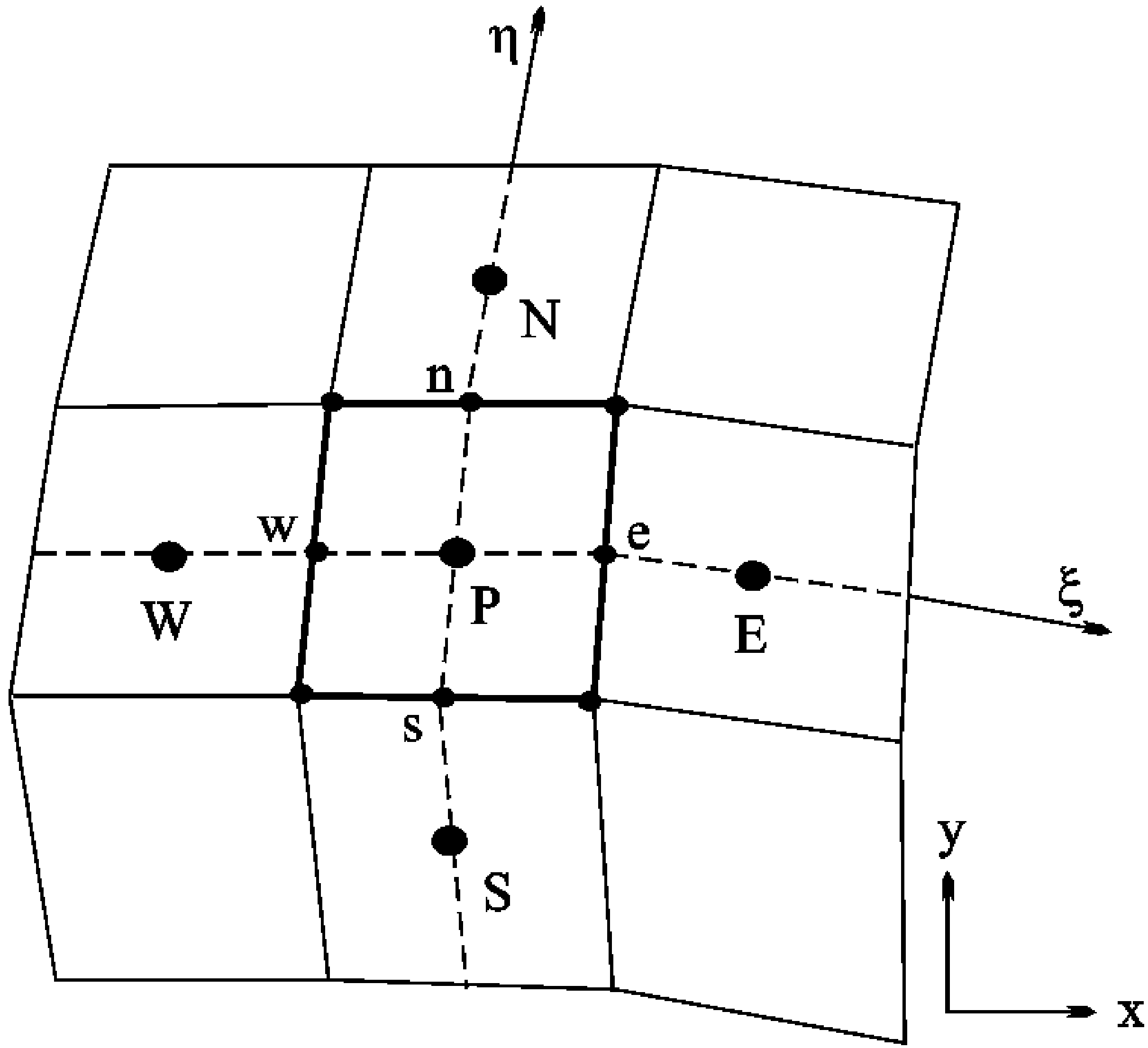
Staggered Grid



Partial Staggered Grid



Non-staggered (Collocated) Grid



Comments

- Staggered Grid is efficient to avoid checkerboard oscillation, but it is more complex in 3D curvilinear grid system
- Partial Staggered Grid is not often used in CFD
- Non-staggered Grid is simpler in 3D curvilinear grid system, but needs to use Rhie and Chow's Momentum Interpolation Technique

Semi-implicit Algorithm (Casulli, 1990)

Use rectangular, staggered grid.

Discretized continuity equation:

$$z_{s,i,j}^{n+1} = z_{s,i,j}^{n} - \frac{\Delta t}{\Delta x}\left(h_{i+1/2,j}^{n} U_{i+1/2,j}^{n+1} - h_{i-1/2,j}^{n} U_{i-1/2,j}^{n+1}\right) - \frac{\Delta t}{\Delta y}\left(h_{i,j+1/2}^{n} V_{i,j+1/2}^{n+1} - h_{i,j-1/2}^{n} V_{i,j-1/2}^{n+1}\right)$$

Discretized momentum equations:

$$U_{i+1/2,j}^{n+1} = F\left(U_{i+1/2,j}^{n}\right) - g\frac{\Delta t}{\Delta x}\left(z_{s,i+1,j}^{n+1} - z_{s,i,j}^{n+1}\right) - \Delta t \gamma_{i+1/2,j}^{n} U_{i+1/2,j}^{n+1}$$

$$V_{i,j+1/2}^{n+1} = F\left(V_{i,j+1/2}^{n}\right) - g\frac{\Delta t}{\Delta x}\left(z_{s,i,j+1}^{n+1} - z_{s,i,j}^{n+1}\right) - \Delta t \gamma_{i,j+1/2}^{n} V_{i,j+1/2}^{n+1}$$

Substituting the above momentum equations to continuity equation yields the Poisson equation for water level.

Projection Method

Use staggered grid.

Discretized momentum equation:

$$\vec{U}^{n+1} = \vec{U}^{n} + \Delta t \vec{G} - \frac{\Delta t}{\rho} \nabla \left(p^{n} + p' \right)$$

$$p = \rho g z_s$$

Define

$$p^{n+1} = p^{n} + p'$$

$$\vec{U}^{*} = \vec{U}^{n} + \Delta t \vec{G} - \frac{\Delta t}{\rho} \nabla p^{n}$$

Thus, pressure and velocity corrections are related by

$$\vec{U}^{n+1} = \vec{U}^{*} - \frac{\Delta t}{\rho} \nabla p'$$

Continuity equation:

$$\frac{\partial h}{\partial t}+\nabla\cdot\left(h\vec{U}^{n+1}\right)=\frac{\partial h}{\partial t}+\nabla\cdot\left(h\vec{U}^{*}\right)-\frac{\Delta t}{\rho}h\nabla^{2}p'-\frac{\Delta t}{\rho}\nabla h\cdot\nabla p'=0$$

Using $$\partial h/\partial t=\left(h^{n+1}-h^{n}\right)/\Delta t=p'/(\rho g\Delta t)$$

and ignoring the last term, one obtains

$$\left(1-\Delta t^{2}gh\nabla^{2}\right)p'=-\Delta t\rho g\nabla\cdot\left(h\vec{U}^{*}\right)$$

Note: the above algorithm is explicit for pressure. An implicit one can be derived by

$$p^{n+1}=p^{*}+p'$$

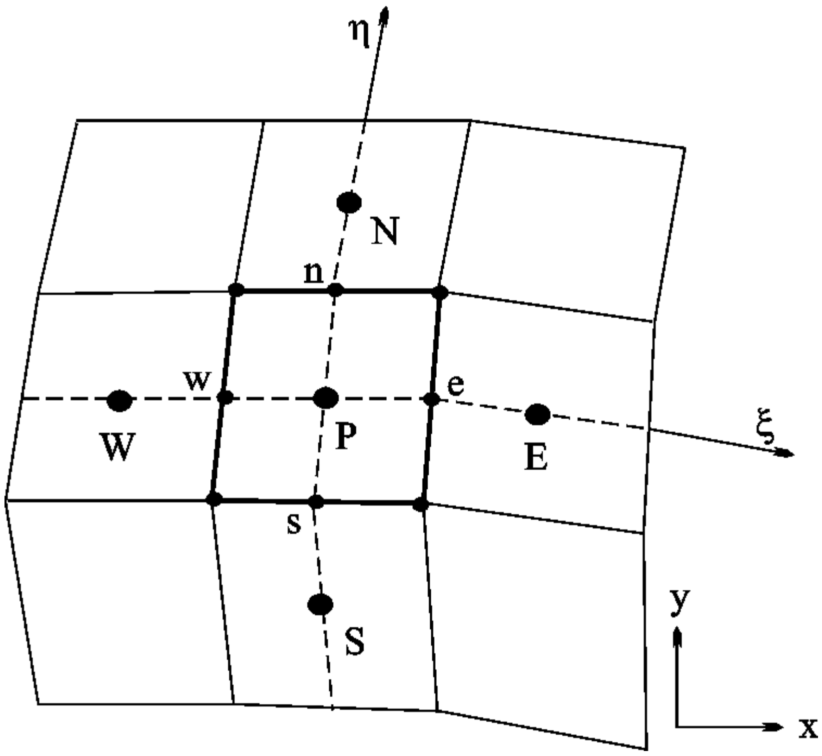
SIMPLE(C) Algorithm (Wu, 2004)

Use quadrilateral, non-staggered grid.

Discretized continuity equation:

$$p_P^{n+1} = p_P^n - g\frac{\Delta t}{\Delta A}\left(F_e - F_w + F_n - F_s\right)$$

where F_e, F_w, F_n and F_s are fluxes at cell faces e, w, n and s.



The key issue is how to evaluate the fluxes F from the quantities stored on nodes P, E, W, N and S.

Linear interpolation may cause oscillations.

Discretized momentum equation at cell center P:

$$U_{i,P}^{n+1} = \frac{1}{a_P^u}\left(\sum_{l=W,E,S,N} a_l^u U_{i,l}^{n+1} + S_{ui}\right) + D_i^2\left(p_s^{n+1} - p_n^{n+1}\right) + \frac{\left(hJ\alpha_i^1 \Delta\eta\right)_P}{a_P^u}\left(p_w^{n+1} - p_e^{n+1}\right)$$

Interpolate the momentum equations discretized on nodes W and P (Rhie and Chow, 1983)

$$U_{i,w}^{n+1} = \left[\left(1 - f_{x,P}\right)G_{i,PW}^1 + f_{x,P}G_{i,P}^1\right] + \left[\left(1 - f_{x,P}\right)/a_{PW}^u + f_{x,P}/a_P^u\right]\left(Jh\alpha_i^1\Delta\eta\right)_w\left(p_W^{n+1} - p_P^{n+1}\right)$$

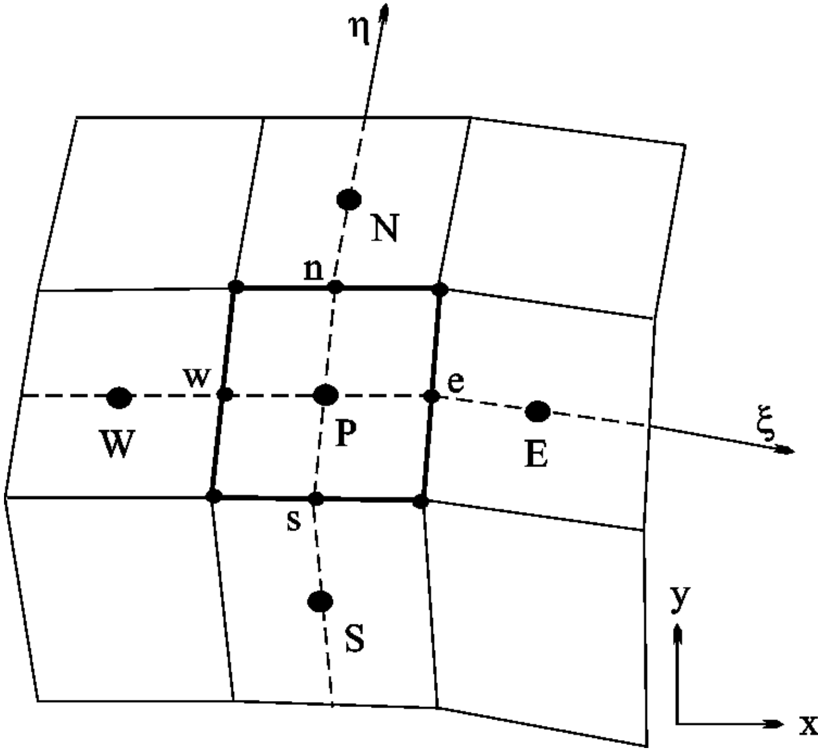
Velocity & pressure corrections:

$$U_{i,w} = U_{i,w}^* + \alpha_u Q_{i,w}^1\left(p_W' - p_P'\right)$$

where $$p' = p^{n+1} - p^*$$

Flux definition leads to:

$$F_w = F_w^* + a_W^p\left(p_W' - p_P'\right)$$



Discretized continuity equation:

$$p_P^{n+1} = p_P^n - g\frac{\Delta t}{\Delta A}\left(F_e - F_w + F_n - F_s\right)$$

Relation of flux and pressure corrections (Rhie and Chow, 1983):

$$F_w = F_w^* + a_W^{(p)}\left(p'_W - p'_P\right) \qquad F_s = F_s^* + a_S^{(p)}\left(p'_S - p'_P\right)$$

Pressure correction equation:

$$\left[\sum_{k=W,E,S,N} a_k^{(p)} + \frac{\Delta A}{g\Delta t}\right] p'_P = \sum_{k=W,E,S,N} a_k^{(p)} p'_k - \left(F_e^* - F_w^* + F_n^* - F_s^*\right) - \frac{\Delta A}{g\Delta t}\left(p_P^* - p_P^n\right)$$

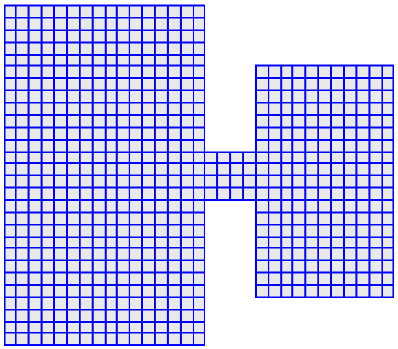
Structured vs. Unstructured Grids

- Structured Grids
  - Rectangular
  - Quadrilateral

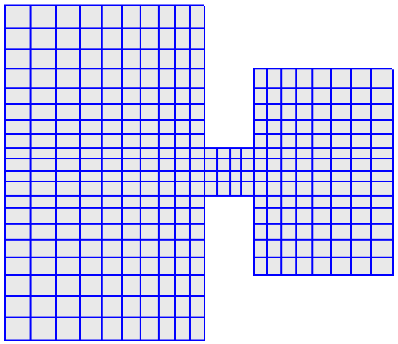
- Unstructured Grids
  - Triangular
  - Quadrilateral with unstructured index and connectivity
  - Polygons

Grids Used

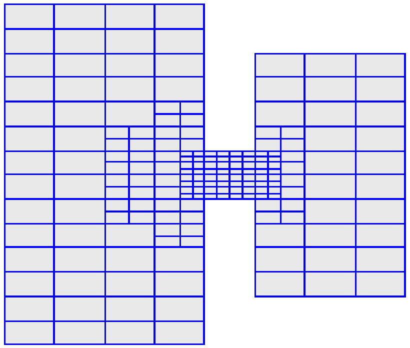
Regular
Cartesian



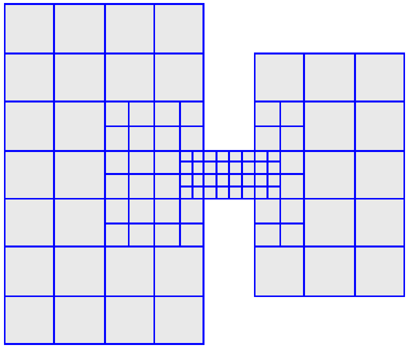
Nonuniform
Cartesian



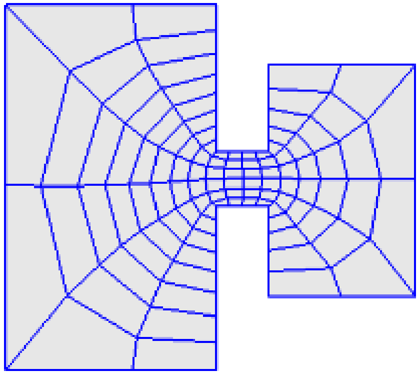
Telescoping
Cartesian



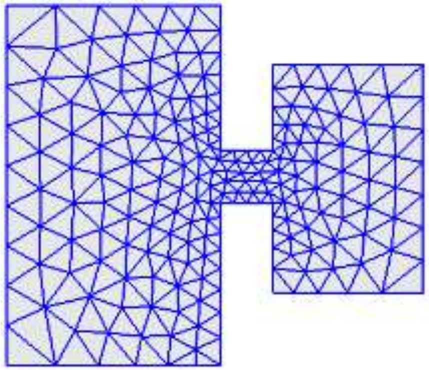
Stretched
Telescoping
Cartesian



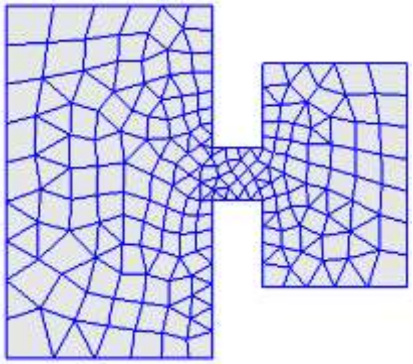
Quadrilateral
(Un)Structured



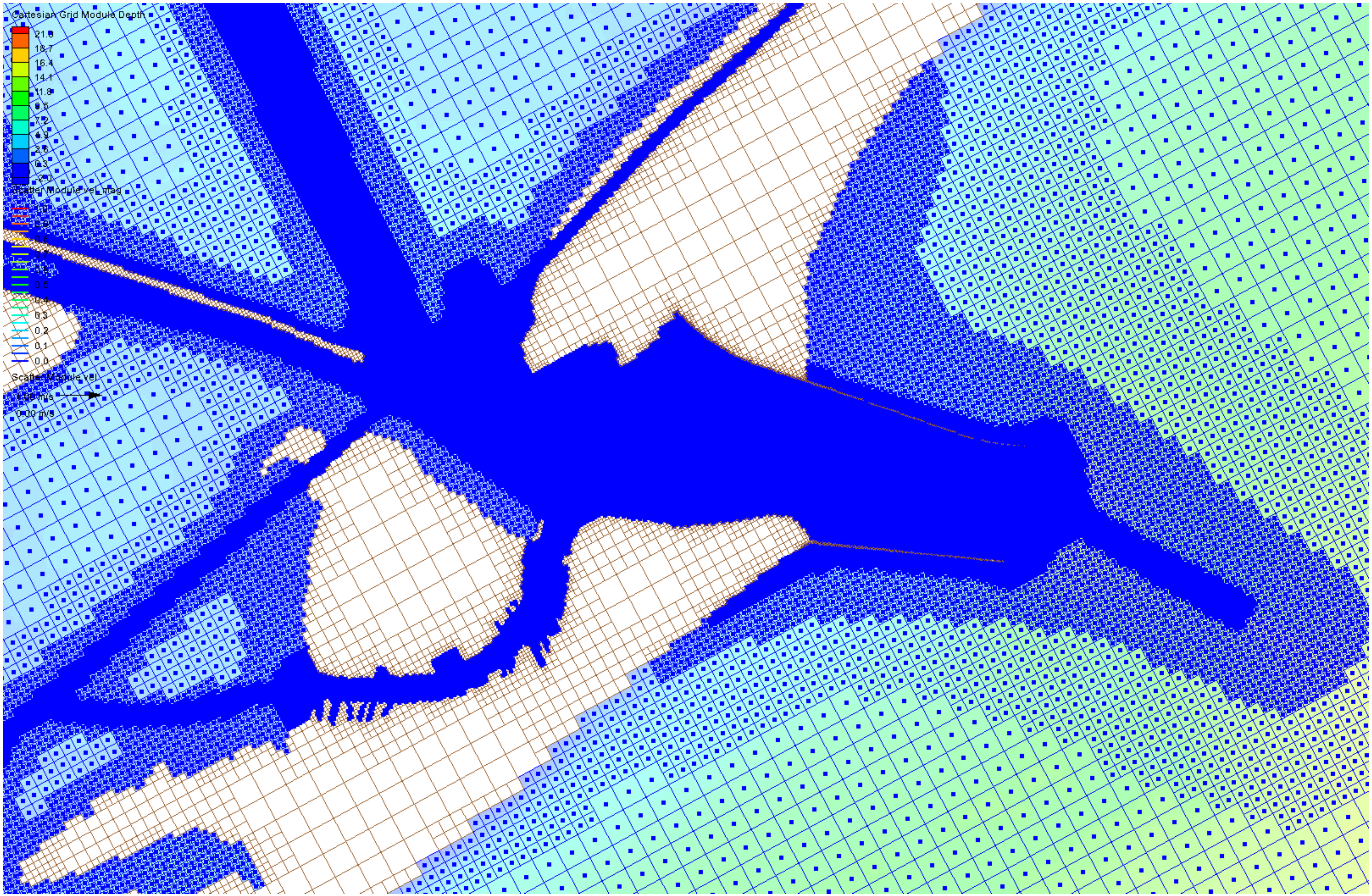
Triangular
Unstructured



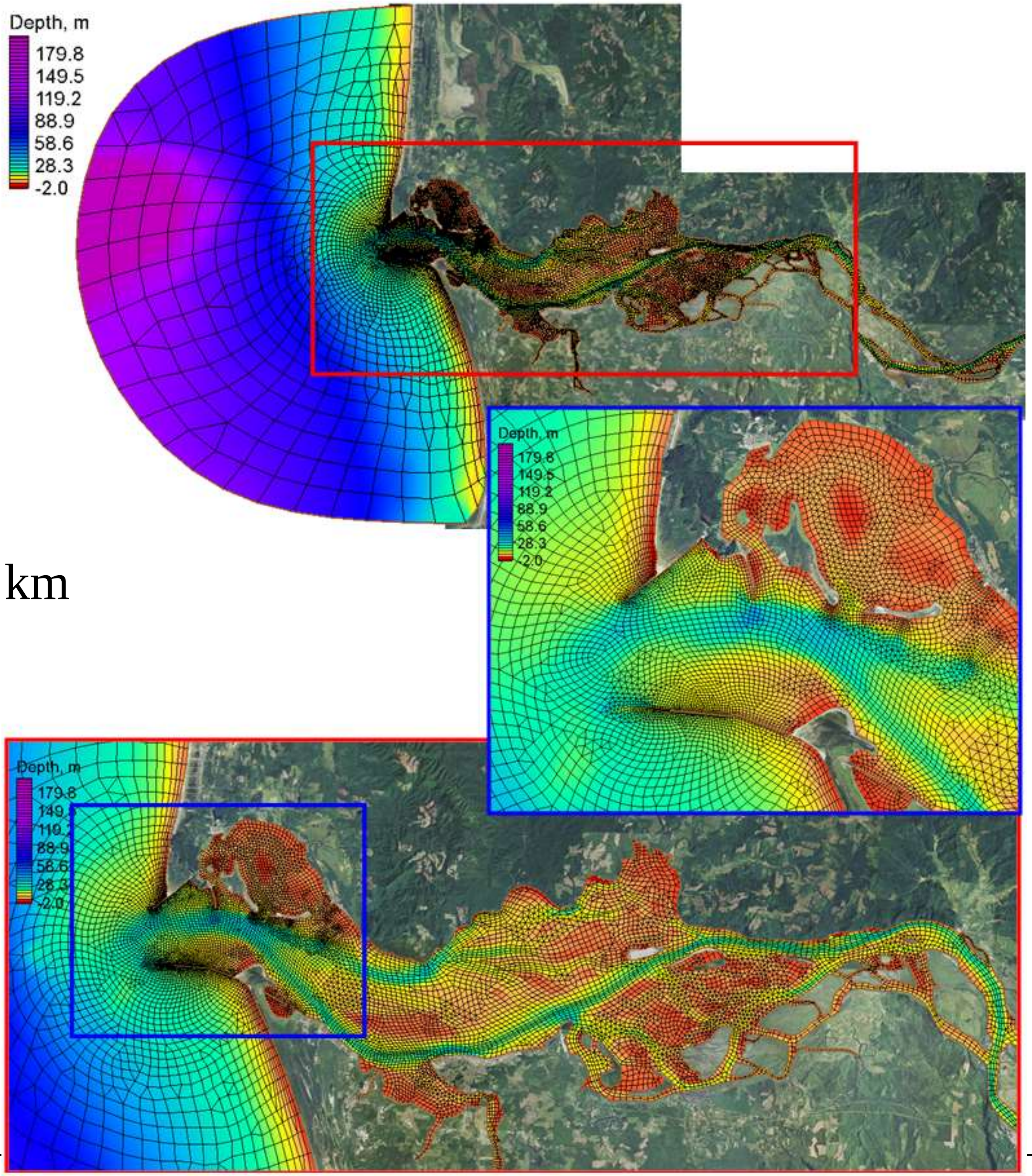
Hybrid
Unstructured



Galveston Entrance Channel, TX



Columbia River, USA



Hybrid mesh

- ~16k cells
- 20 m to 3.5 km resolution

Comments

	Structured	Unstructured
Grid Connectivity	Simpler	More complex
Grid Flexibility	Less	More
Idle Nodes	More	Less
Coefficient Matrix	Banded, Symmetric	Sparse, Asymmetric
Algebraic Eq. Solver	More efficient	Less efficient
Efficiency	Problem-dependent	Problem-dependent

Explicit vs. Implicit Schemes

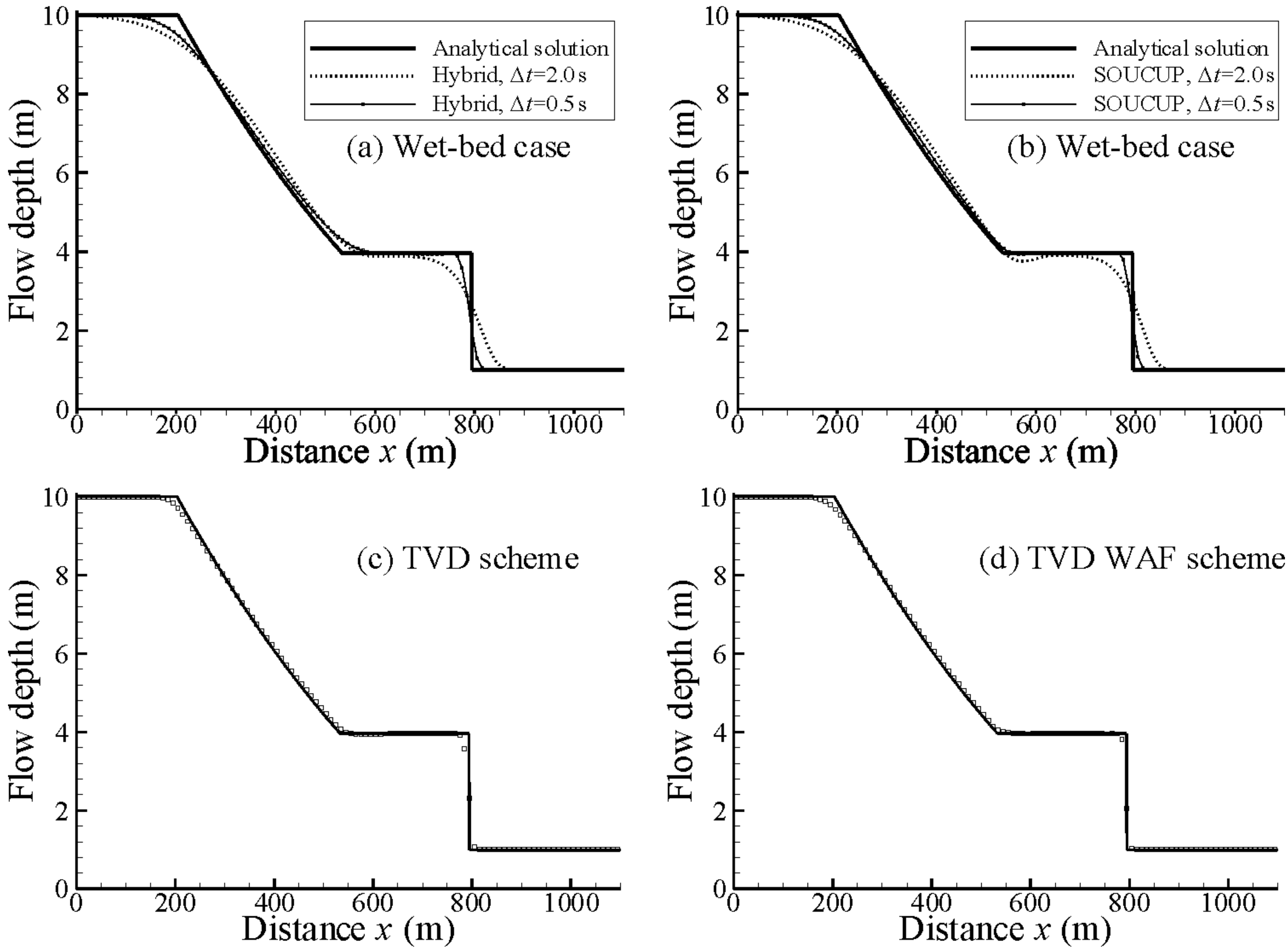
- Explicit
  - Euler Scheme
  - Runge-Kutta Method

- Alternate Direction Implicit
- Operator Splitting Method

- Full-Domain Implicit
  - Backward Difference (Two-Level)
  - Three-level Implicit
  - Semi-Implicit (e.g. Crank-Nicholson)

Comments

	Explicit	Implicit
Coding	Simpler	More complex
Parallel	Easier	More difficult
Drying and wetting	Simpler	More complex
Numerical diffusion	Less	More
Time step	Shorter	Longer
Efficiency (single-processor computer)	Less	More (depend on iteration solver)



Dam-Break Flows: (a)&(b) Implicit; (c)&(d) Explicit

Explicit vs. Implicit Schemes

- For highly transient flows such as dam-break flow, explicit algorithms are usually more accurate.

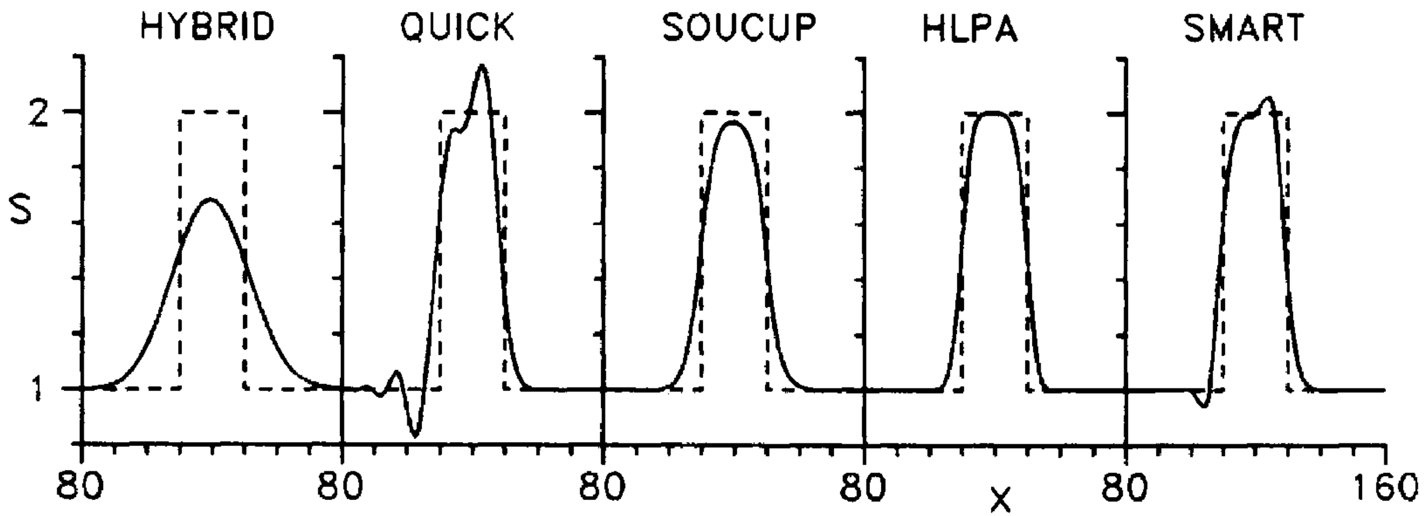
- If there is sharp gradient, explicit algorithms are usually more accurate.

- For gradually varying flows and mass transport, numerical diffusion by implicit schemes is usually acceptable.

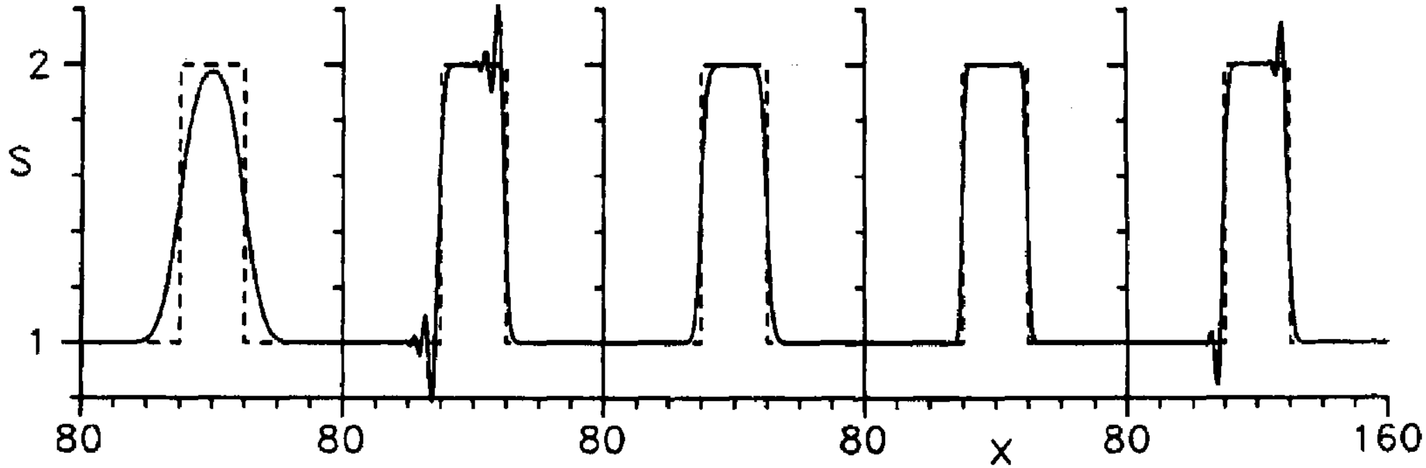
Upwinding Schemes

- Hybrid Upwind/Central Difference
- Exponential Difference Scheme
- QUICK (with limiters)
- SOUCUP and HLPA (used in FVM)
- Upwind FEM Schemes
- Others

Upwinding Schemes in Case of Pure Advection (Zhu, 1991)



(c) Predicted profiles at t = 100 (201×2 grid, $\Delta t = 0.4$)



(d) Predicted profiles at t = 100 (1001×2 grid, $\Delta t = 0.1$)

Algebraic Equation Solvers

- Point-by-Point Methods
  - Jacobi Method
  - Gauss-Seidel Method
- Line-by-Line Methods
  - ADI
- SIP (Strongly Implicit Procedure)
- Conjugate Gradient Methods
  - CG, CGS, CGSTAB, GMRES
  - With Preconditioning (e.g. ICCG)

Comments

- Jacobi and Gauss-Seidel Methods
  - For both structured and unstructured grids
  - Easy to be parallelized

- ADI and SIP methods
  - For structured grids
  - Efficient

- CG, CGS, CGSTAB, GMRES
  - For structured and/or unstructured grids
  - Some are efficient

Efficiency of Algebraic Equation Solvers (Ferziger and Peric, 1995)

Grid	GS	LGS-ADI	SIP	ICCG
8x8	74	5	4	4
16x16	293	8	6	6
32x32	1164	18	13	11
64x64	4639	53	38	21
128x128	--	189	139	41

Numbers of iterations required by various solvers to reduce the normalized L_1 residual norm below 10^{-5} for the 2D Laplace equation with Direchlet boundary conditions on a rectangular domain 10x1 with uniform grid in both directions.

Under-Relaxation -- Traditional

Consider

$$a_P \phi_{i,j} = a_W \phi_{i-1,j} + a_E \phi_{i+1,j} + a_S \phi_{i,j-1} + a_N \phi_{i,j+1} + b$$

$$\Rightarrow \quad A\Phi = b$$

Define $\Delta\Phi = \Phi^{(1)} - \Phi^{(0)}$ $\qquad R = b - A\Phi^{(0)}$

$$\Rightarrow \quad A\Delta\Phi = R$$

Solve it and apply under-relaxation:

$$\Phi^{(1)} = \Phi^{(0)} + \alpha_\phi \Delta\Phi$$

Under-Relaxation (Majumdar 1988)

Reformulate $$\phi_P = \left(\sum_{k=W,E,S,N} a_k \phi_k + b\right) \Big/ a_P$$

Apply under-relaxation:

$$\phi_P^{(1)} = \alpha_\phi \left(\sum_{k=W,E,S,N} a_k \phi_k + b\right) \Big/ a_P + (1-\alpha_\phi)\phi_P^{(0)}$$

$$\Rightarrow \quad \frac{a_P}{\alpha_\phi}\phi_P^{(1)} = \sum_{k=W,E,S,N} a_k \phi_k + b + \frac{1-\alpha_\phi}{\alpha_\phi} a_P \phi_P^{(0)}$$

Then $A'\Phi = b'$ $\Rightarrow$ $A'\Delta\Phi = R'$

$$\Rightarrow \quad \Phi^{(1)} = \Phi^{(0)} + \Delta\Phi$$

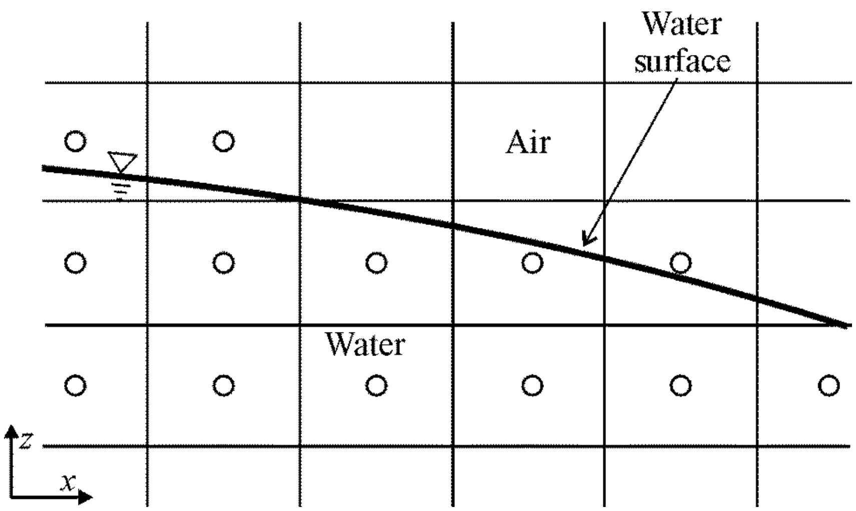
Majumdar (1988) used this in SIMPLE(C) on collocated grid with Rhie and Chow's momentum interpolation.

Wetting and Drying

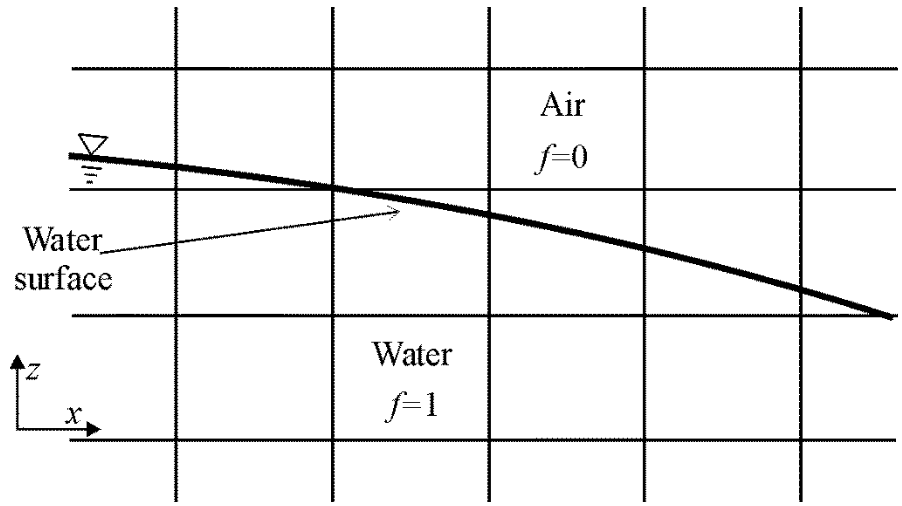
- Problem due to water edge change.
- Dry nodes are excluded in explicit algorithms.
- Dry nodes are included in implicit algorithms, but treated with
  - Small imaginary depth;
  - “Freezing” method;
  - Porous medium method; or
  - Finite slot method.

Free Surface

- Volume tracking methods:
  - MAC
  - VOF
  - Level set



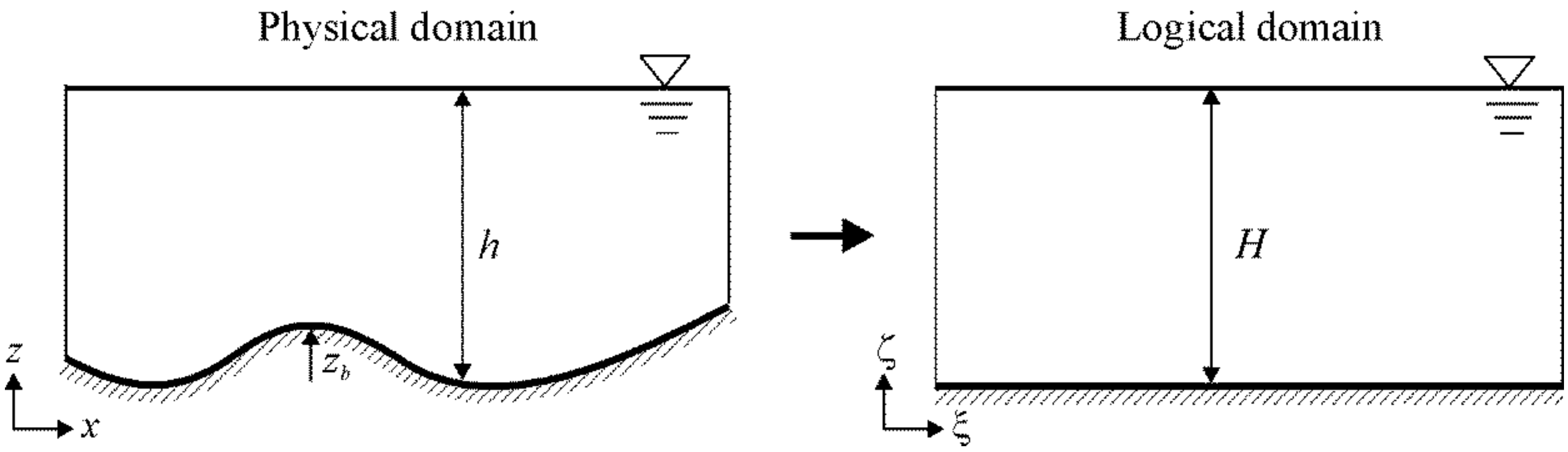
MAC method



VOF method

- Surface tracking methods:

Moving or Adaptive Mesh



Stretching or σ Coordinate.

Free Surface Calculations

(1) Kinematic condition

$$\frac{\partial z_s}{\partial t} + u_{hx}\frac{\partial z_s}{\partial x} + u_{hy}\frac{\partial z_s}{\partial y} = u_{hz}$$

(2) Depth-integrated 2-D continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial (hU_x)}{\partial x} + \frac{\partial (hU_y)}{\partial y} = 0$$

(3) Horizontal 2-D Poisson equation (Wu et al., 2000)

$$\frac{\partial^2 z_s}{\partial x^2} + \frac{\partial^2 z_s}{\partial y^2} = \frac{S_z}{g}$$

Valid only for gradually-varied flows

with $$S_z = -\frac{\partial}{\partial t}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\right) - \left(\frac{\partial U_x}{\partial x}\right)^2 - 2\frac{\partial U_x}{\partial y}\frac{\partial U_y}{\partial x} - \left(\frac{\partial U_y}{\partial y}\right)^2 - U_x\left(\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_y}{\partial x \partial y}\right) - U_y\left(\frac{\partial^2 U_x}{\partial x \partial y} + \frac{\partial^2 U_y}{\partial y^2}\right)$$
$$+ \frac{1}{\rho}\left(\frac{\partial^2 T_{xx}}{\partial x^2} + 2\frac{\partial^2 T_{xy}}{\partial x \partial y} + \frac{\partial^2 T_{yy}}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial}{\partial x}\left(\frac{\tau_{bx}}{h}\right) - \frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{\tau_{by}}{h}\right)$$

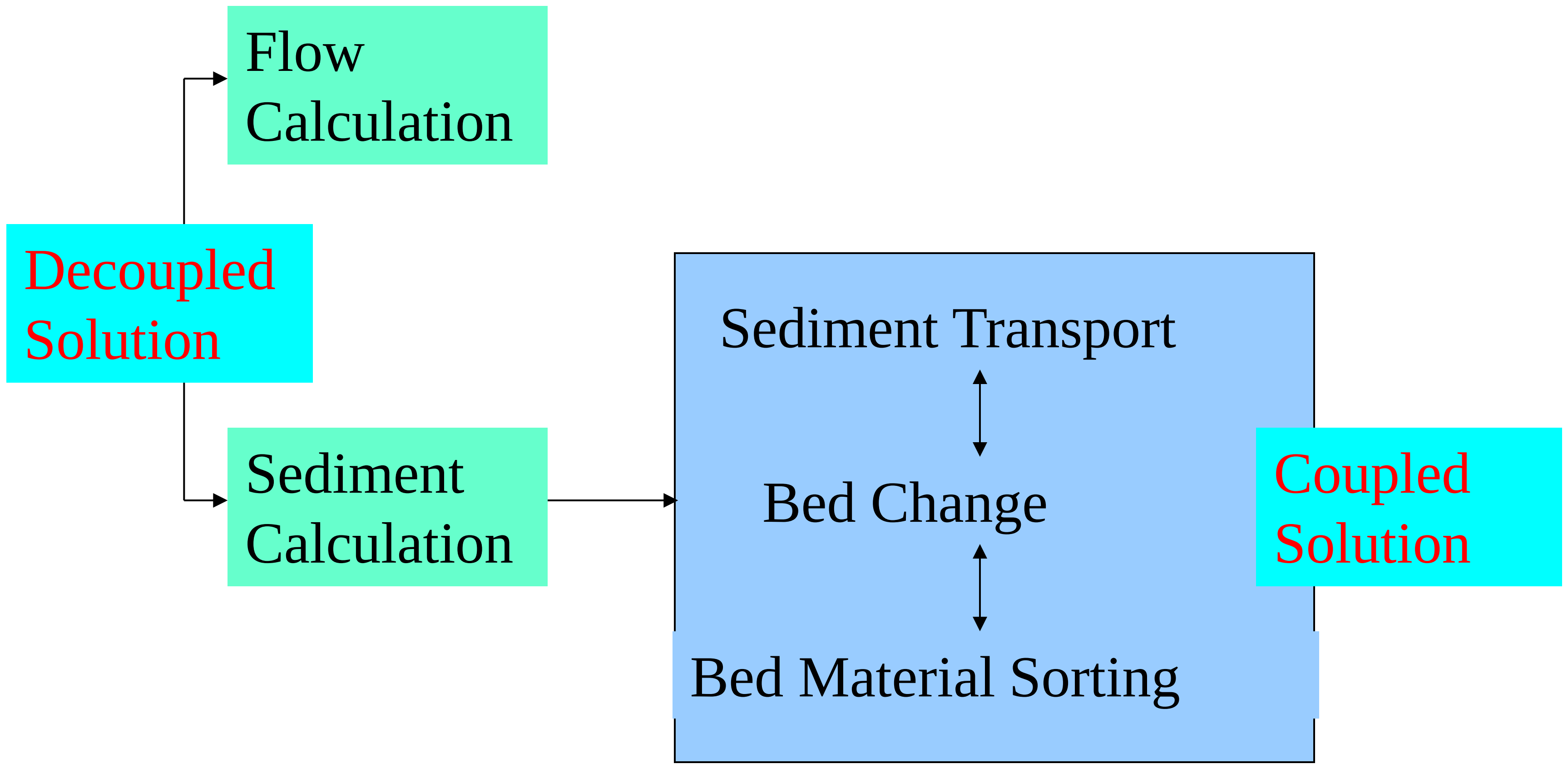
Coupling Flow and Sediment

- Fully coupling
  - Consider interactions between flow, sediment transport and bed change
    - Holly et al. (1990): fully implicit
    - Recent models of dam-break flow over mobile beds (Wu et al. 2007, 2012; Cao et al., 2004): fully explicit
  - Valid for high-speed flow with high concentration

- Fully decoupling
  - Calculate flow, sediment transport, bed change and bed material sorting separately
  - Valid for common flows with low sediment concentration
    - Most of existing models use this approach

- Semi-coupling
  - Wu et al. (2004)

Semi-Coupling Technique



Discretization: Sediment Transport

- Suspended-load transport equation

$$\frac{\rho_P \Delta A_P}{\Delta t}\left(\frac{h_P^{n+1} C_{k,P}^{n+1}}{\beta_{s,P}^{n+1}} - \frac{h_P^n C_{k,P}^n}{\beta_{s,P}^n}\right) = a_E^{(C)} C_{k,E}^{n+1} + a_W^{(C)} C_{k,W}^{n+1} + a_N^{(C)} C_{k,N}^{n+1} + a_S^{(C)} C_{k,S}^{n+1} - a_P^{(C)} C_{k,P}^{n+1}$$

$$+ \alpha \omega_{sk} \rho_P \Delta A_P \left(C_{*k,P}^{n+1} - C_{k,P}^{n+1}\right) + S_{k,P}$$

$(k=1, 2, \ldots, N)$

- Bed-load transport equation

$$\frac{\Delta A_P}{\Delta t}\left(\frac{q_{bk,P}^{n+1}}{u_{b,P}^{n+1}} - \frac{q_{bk,P}^{n}}{u_{b,P}^{n}}\right) = a_E^{(q)} q_{bk,E}^{n+1} + a_W^{(q)} q_{bk,W}^{n+1} + a_N^{(q)} q_{bk,N}^{n+1} + a_S^{(q)} q_{bk,S}^{n+1} - a_P^{(q)} q_{bk,P}^{n+1}$$

$$+ \frac{\Delta A_P}{L_t}\left(q_{b*k,P}^{n+1} - q_{bk,P}^{n+1}\right)$$

- Bed change equation

$$\Delta z_{bk,P}^{n+1} = \frac{\alpha \omega_{sk} \Delta t}{1 - p'_m} \left(C_{k,P}^{n+1} - C_{*k,P}^{n+1} \right) + \frac{\Delta t}{(1 - p'_m) L_t} \left(q_{bk,P}^{n+1} - q_{b*k,P}^{n+1} \right)$$

$$\Delta z_{b,P}^{n+1} = \sum_{k=1}^{N} \Delta z_{bk,P}^{n+1}$$

- Bed material sorting equation

$$p_{bk,P}^{n+1} = \frac{\Delta z_{bk,P}^{n+1} + \delta_{m,P}^{n} p_{bk,P}^{n} + p_{bk,P}^{*n} \left(\delta_{m,P}^{n+1} - \delta_{m,P}^{n} - \Delta z_{b,P}^{n+1} \right)}{\delta_{m,P}^{n+1}}$$

- Sediment transport capacity

$$C_{*k,P}^{n+1} = p_{bk,P}^{n+1} C_{k,P}^{*n+1} \qquad q_{b*k,P}^{n+1} = p_{bk,P}^{n+1} q_{bk,P}^{*n+1}$$

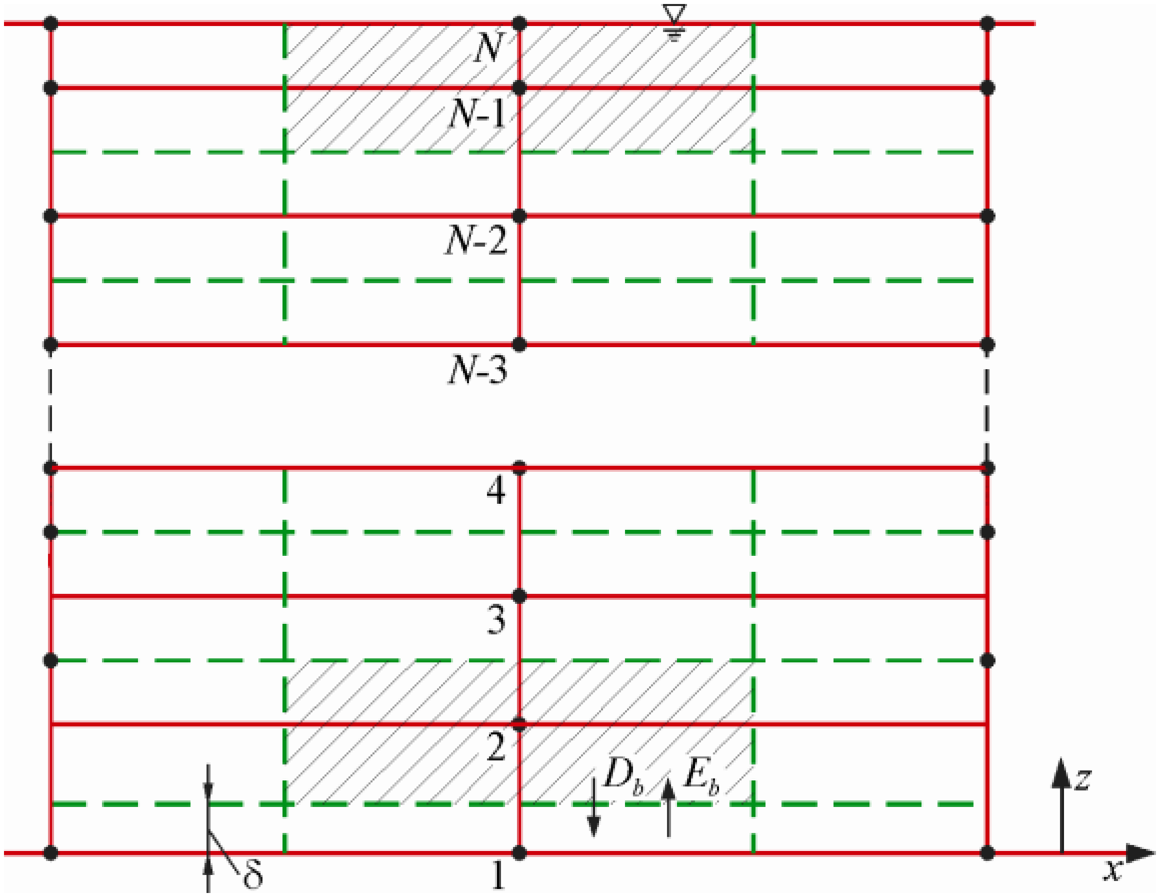
Solution of Sediment Transport

- Coupling bed change and bed material sorting equations and sediment transport capacity formulas yields

$$\Delta z_{b,P}^{n+1} = \left\{ \sum_{k=1}^{N} \frac{\alpha\omega_{sk}\Delta t \delta_{m,P}^{n+1} C_{k,P}^{n+1} + \Delta t \delta_{m,P}^{n+1} q_{bk,P}^{n+1} / L_t}{(1 - p'_m)\delta_{m,P}^{n+1} + \alpha\omega_{sk}\Delta t C_{k,P}^{*n+1} + \Delta t q_{bk,P}^{*n+1} / L_t} - \sum_{k=1}^{N} \frac{\left[\alpha\omega_{sk}\Delta t C_{k,P}^{*n+1} + \Delta t q_{bk,P}^{*n+1} / L_t\right]\left[\delta_{m,P}^{n} p_{bk,P}^{n} + \left(\delta_{m,P}^{n+1} - \delta_{m,P}^{n}\right) p_{bk,P}^{*n}\right]}{(1 - p'_m)\delta_{m,P}^{n+1} + \alpha\omega_{sk}\Delta t C_{k,P}^{*n+1} + \Delta t q_{bk,P}^{*n+1} / L_t} \right\} \Bigg/ \left\{ 1 - \sum_{k=1}^{N} \frac{\left[\alpha\omega_{sk}\Delta t C_{k,P}^{*n+1} + \Delta t q_{bk,P}^{*n+1} / L_t\right] p_{bk,P}^{*n}}{(1 - p'_m)\delta_{m,P}^{n+1} + \alpha\omega_{sk}\Delta t C_{k,P}^{*n+1} + \Delta t q_{bk,P}^{*n+1} / L_t} \right\}$$

- Thus, a coupled solution procedure is established for sediment. This procedure is stable and avoids negative bed material gradation.

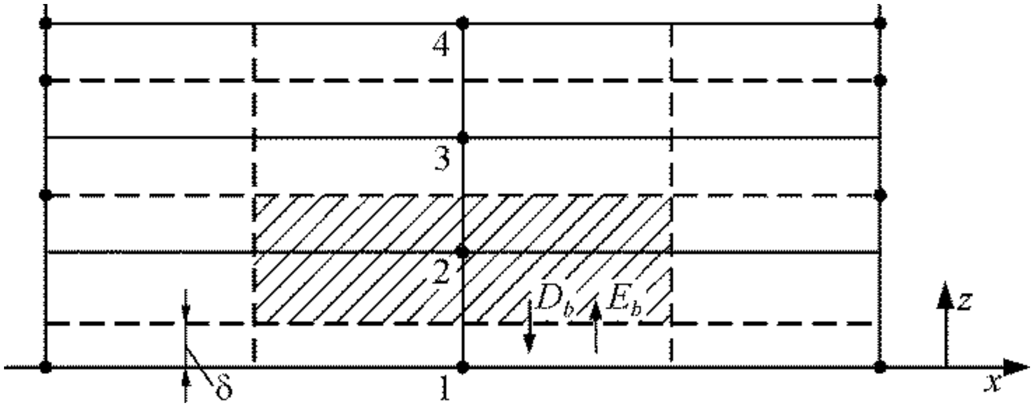
3-D Flow and Sediment Meshes



Note that the flow and sediment control volumes near the bed are not conformal, because suspended-load domain starts from the interface between bed load and suspended load, whereas the flow domain starts from the bed. One simple treatment is to set the bed-load layer covering the first near-bed layer and the suspended-load domain starts from the second layer.

Near-bed Sediment Flux

Assume the sediment concentration distribution between the interface δ and the cell center 2 to be the same as that at equilibrium. Ignoring the storage, convection and horizontal diffusion in the suspended-load transport equation yields



$$\frac{\partial}{\partial z}\left(\varepsilon_s \frac{\partial c_k}{\partial z} + \omega_{sk} c_k\right) = 0$$

which has the locally-linearized analytical solution:

$$c_k = a_1 + a_2 e^{-z\omega_{sk}/\varepsilon_s}$$

Using the sediment concentrations at δ and point 2 as conditions to determine the coefficients a_1 and a_2 :

$$c_{bk} = c_{2k} + c_{b*k}[1 - e^{-(z_2 - z_b - \delta)\omega_{sk}/\varepsilon_s}]$$

In the case that z_2-z_b-δ is small, the above exponential scheme can be simplified as the linear scheme:

$$c_{bk} = c_{2k} + c_{b*k}(z_2 - z_b - \delta)\frac{\omega_{sk}}{\varepsilon_s}$$

Publications Related

W. Wu (2004). “Depth-averaged 2-D numerical modeling of unsteady flow and nonuniform sediment transport in open channels,” J. Hydraulic Eng., ASCE, 130(10), 1013–1024.

W. Wu, A. Sanchez, and M. Zhang (2011). “An implicit 2-D shallow water flow model on unstructured quadtree rectangular mesh.” Journal of Coastal Research, Special Issue, No. 59, pp. 15–26.

W. Wu and Q. Lin (2011). “An implicit 3-D finite-volume coastal hydrodynamic model.” Proc., 7th Int. Symposium on River, Coastal and Estuarine Morphodynamics, September 6-8, Beijing, China.

W. Wu (2007), Computational River Dynamics, Taylor & Francis, UK, 494 p.